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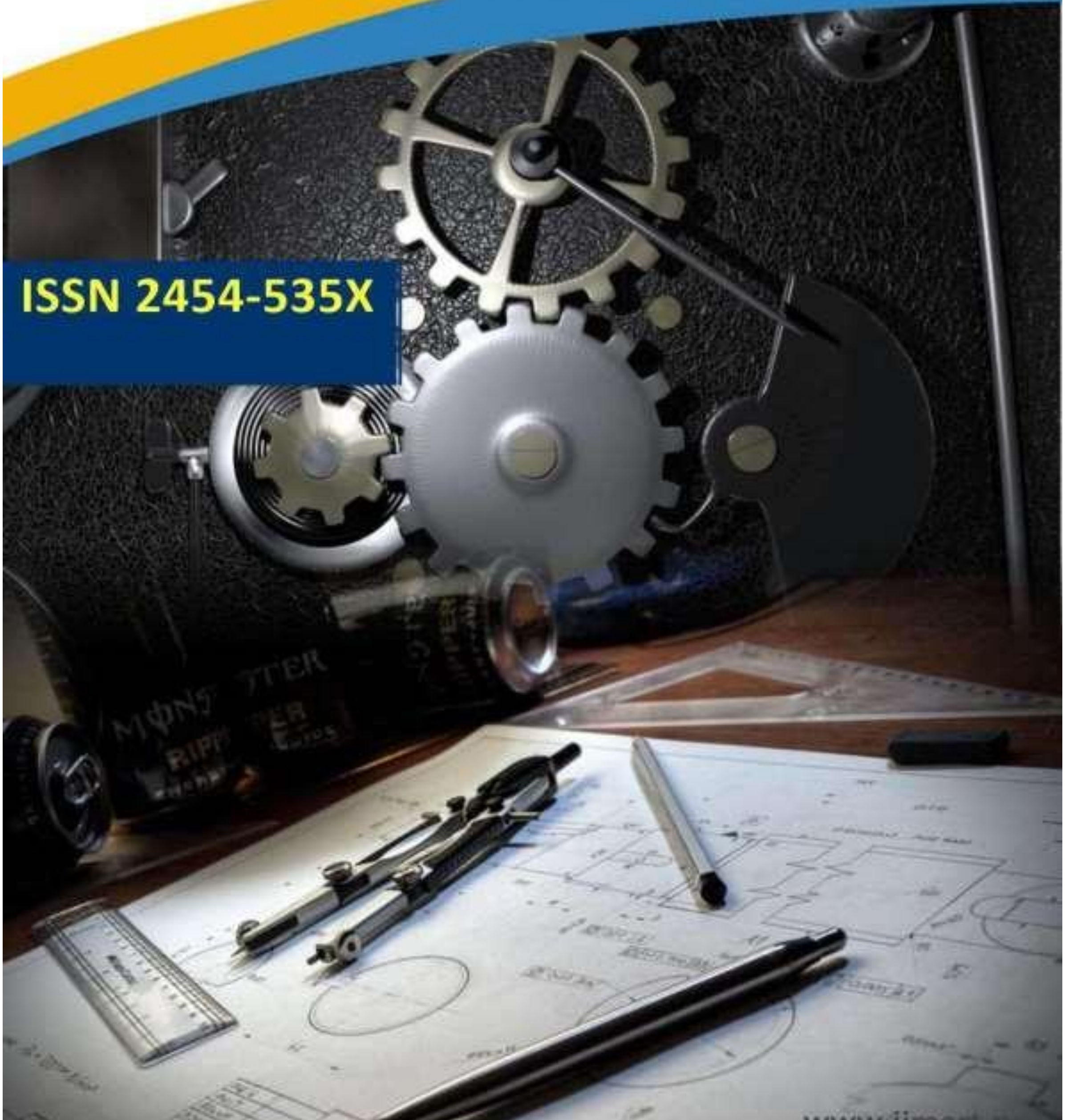

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Seamless Immersed Boundary Method for Flow Simulation with Heat Transfer on Dual-Resolution Grid

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ABSTRACT:
In this study, we suggested a simulation approach for the flow with heat transfer that combines the dual-resolution grid method and the seamless immersed boundary method (SIBM). When applying SIBM to the flow simulation with heat transfer, a finer grid is used than in the flow simulation without heat transfer since the energy equation requires a higher grid resolution than the Navier- Stokes equation. Particularly, solving the pressure equation at the grid resolution required by the energy equation results in a considerable loss of computing efficiency because the majority of the computational time is often used to solve the pressure equation in the incompressible flow. The Navier-Stokes equations and the energy equations are solved at different speeds in the current method.

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KEYWORDS: simulation; dual-resolution grid; seamless immersed boundary method (SIBM

INTRODUCTION

Computational fluid dynamics does a good job of handling the flow with heat transport issue, which has significant engineering implications. The flow inside a computer's enclosure is an illustration of flow with heat transfer, and cooling the CPU is crucial for the computer to run steadily. However, since computers have recently become smaller, their internal architecture has complicated, necessitating the need for more effective cooling. There are numerous components with complex shapes in the computational area of the flow simulation within the computer housing.The immersed boundary method (IBM) [1], a Cartesian grid methodology, has gained popularity in recent years for simulating flow around objects of complex shape. Within IBM,

SIBM ON DUAL-RESOLUTION GRID

Governing Equations

The governing equations are the continuity equation, the incompressible Navier-Stokes equations and the energy equation. The forcing term and heat flux are added to the Navier-Stokes equation and the energy equation for the SIBM. The non-dimensional governing equations are written as,

$$\frac{\partial\partial u_{ii}}{\partial\partial x_{ii}} = 0, \qquad (1)$$

$$\frac{\partial\partial u_{ii}}{\partial\partial t} = F_{ii} - \frac{\partial\partial p}{\partial\partial x_{ii}} + G_{ii}, \qquad (2)$$

$$\frac{\partial\partial T}{} = F + QQ. \qquad (3)$$

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$\partial\partial t \quad T$

The last term in Eq. (2), (3), G_{ii} and QQ denote the additional forcing term and heat flux term for the SIBM. F_{ii} and F_T denote the convective and diffusion terms in Eq. (2), (3).

$$F_{ii} = -u_{jj}\frac{\partial\partial u_{ii}}{\partial\partial x_{jj}} + \frac{1}{Re}\frac{\partial\partial^2 u_{ii}}{\partial\partial x_{jj}\partial\partial x_{jj}} \tag{4}$$

$$F_T = -u_{jj}\frac{\partial\partial T}{\partial\partial x_{jj}} + \frac{1}{Pr\cdot Re}\frac{\partial\partial^2 T}{\partial\partial x_{jj}\partial\partial x_{jj}} \tag{5}$$

Where, Re denotes the Reynolds number defined by $Re = L_0U_0/\nu_0$ and Pr denotes the Prandtl number defined by $Pr = \nu_0/\alpha_0$. L_0, U_0, ν_0 and α_0 are the reference length, the reference velocity, the kinematic viscosity and the thermal diffusivity, respectively.

Numerical Method

The incompressible Navier-Stokes equations and the energy equation, i.e. Eq. (2), (3), are solved by the finite difference method on the collocated grid arrangement. The convective, diffusion and pressure terms are discretized by the conventional second order centered finite difference method. The time derivative terms are discretized by the forward Euler method. For the time integration of the Navier-Stokes equations, the fractional step approach [6] based on the forward Euler method is applied. For the incompressible Navier-Stokes equations in the IBM, the fractional step approach can be written by where u_{ii}^* denotes the fractional step velocity and Δt is the time increment. The resulting pressure equation is solved by the SOR method.

$$u^* = u^n + \Delta t\, F^n{}_{,ii} \quad (6)$$

$$u^{n+1} = u^* + \Delta t\left(-\frac{\partial\partial p^n}{\partial\partial x_{ii}} + G^n_{ii}\right), \quad (7)$$

Seamless Immersed Boundary Method

In order to apply the SIBM to the flow simulation with heat transfer, the additional forcing terms and heat flux term in the Navier-Stokes equations and the energy equation should be estimated. In this study, the direct forcing term estimation [7] is adopted for both the forcing terms and the heat flux term. The direct forcing term estimation for the forcing terms in the SIBM is shown in Fig. 1. In the figure, I, J are the grid index. The estimation for the heat flux term in the isothermal condition is the same as for the forcing terms. The forcing terms and heat flux term can be determined by

$$G^n_{ii} = -F^n_{ii} + \frac{\partial p}{\partial\partial x_{ii}} + \frac{\tilde{u}_i^{n+1} - u^n_{ii}}{\Delta t}, \tag{8}$$

$$QQ = -F_T + \frac{\tilde{T}^{n+1} - T^n}{\Delta t}, \tag{9}$$

where $\tilde{u}_{ii}^{n+1}$ and $\tilde{T}^{n+1}$ are predicted velocity and temperature that satisfy the velocity and temperature conditions on the virtual boundary. At grid points in the fluid media adjacent to the virtual boundary, the predicted velocity and temperature are linearly interpolated from boundary conditions and surrounding values. At grid points in the solid media, these are determined from the velocity and temperature conditions at that grid point.

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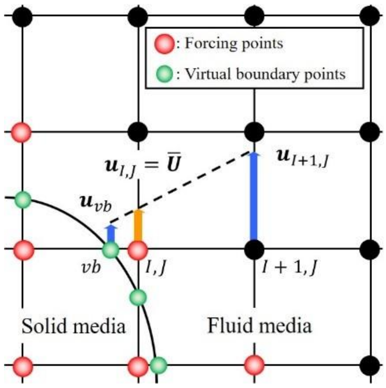

Figure 1. Forcing points and direct forcing estimation in the SIBM

Dual-Resolution Grid

In this study, the Navier-Stokes equations and energy equations are solved at different resolutions by using the dual-resolution grid in order to perform efficiently the flow simulation with heat transfer. In the present method,the resolution of the energy equation is set to twice that of the Navier-Stokes equations. In this grid resolution condition, the positions of velocity, pressure and temperature components are defined as seen in Fig 2. When applying the dual-resolution grid method to the simulation for two-phase flows, the flow variables are generallydefined in a staggered arrangement [4], [5], [8]. However, all flow variables are defined at cell center in this study because collocated grid are generally adopted in the flow simulation by the SIBM. In this case, the velocity component for the advective term in the energy equation is interpolated from the surrounding grid points because the positions of velocity components and temperature component are different.

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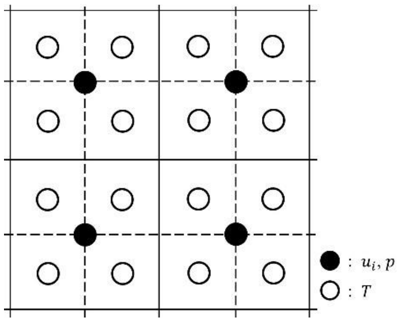

Figure 2. The positions of velocity, pressure and temperature components on the dual-resolution grid

FLOW AROUND a 2-DIMENSIONAL HEATED CIRCULAR CYLINDER

In order to validate the SIBM on the dual-resolution grid, the flow around a 2-dimensional heated circular cylinder is considered. The computational domain is shown in Fig. 3. The reference length is a diameter of the circular cylinder D. The impulsive start determined by the uniform flow ($u = 1$, $v = 0$, $T = 0$) is adopted. On the inflow boundary, the velocity and temperature are fixed by the uniform flow and the pressure is imposed by the Neumann condition obtained by the normal momentum equation. The velocity and temperature are extrapolated from the inner points and the pressure is obtained by the Sommerfeld radiation condition [9] on theoutflow and side boundaries. On the virtual boundary and inside the boundary, the non-slip ($u = 0$, $v = 0$) and the isothermal ($T = 1$) conditions are imposed. The Reynolds number is set as $Re = 200$, 218 and the Prandtl number is set as $Pr = 0.717$ according to references [10, 11].

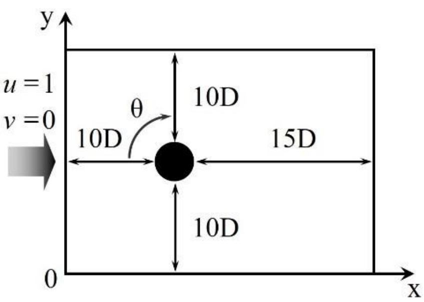

Figure 3. Computational domain

First, the influence by the grid resolution on the results on the single-resolution grid is investigated. The computational grid is the hierarchical Cartesian grid that is fine near the circular cylinder. The grid resolution near the circular cylinder is $\Delta = D/40$, $D/80$, $D/160$ and $D/320$, respectively. In Table 1, the time-averaged drag coefficient, the amplitude of lift coefficient and the Strouhal number in $Re = 200$ are shown with the reference results [10]. The drag and lift coefficients (C_D and C_L) are determined by

$$C_D = \frac{-\int \left(G_{00}\delta_{x} - u_i \frac{\partial u}{\partial x_i} - \frac{\partial u}{\partial t} \right)}{\frac{1}{2}\rho_0 U_0^2 D}, \quad (10)$$

$$C_L = \frac{-\int \left(G_{00}\delta_{y} - u_i \frac{\partial v}{\partial x_i} - \frac{\partial v}{\partial t} \right)}{\frac{1}{2}\rho_0 U_0^2 D}, \quad (11)$$

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where O denotes the region to which the forcing term is added in the SIBM. The results in each grid are in good agreement the reference ones. The respective characteristic quantities are in good agreement at grid resolutions from $D/80$ to $D/320$. Figure 4 shows the time averaged local Nusselt number on the circular cylinder surface in $Re = 218$ with the reference results [11]. The local Nusselt number on the circular cylinder surface is determined by

$$Nu(\theta\theta) = -\frac{D}{T_{vb} - T_{\infty}} \frac{\partial\partial T(\theta\theta)}{\partial\partial r}, \tag{12}$$

where T_{vb} denotes the temperature on the virtual boundary of the circular cylinder and $\partial\partial T(\theta\theta)/\partial\partial r$ denotes the temperature gradient of normal direction. The local Nusselt numbers in the fine grid is closer to the reference ones. The local Nusselt numbers are in good agreement when the grid resolution is $D/160$ and $D/320$. In the grid resolution $D/80$, it is not agreement it in the grid resolution $D/160$ or $D/320$. As a result, in the present method using the dual-resolution grid, the grid resolution near the circular cylinder is set to $D/80$ for the Navier-Stokes equations and $D/160$ for the energy equation [12-15].

Table 1. Comparison of characteristic quantities ($Re = 200$)

	C_D	$C_{L_{amp}}$	St
$\Delta = D/40$	1.322	0.629	0.199
$\Delta = D/80$	1.352	0.678	0.198
$\Delta = D/160$	1.358	0.687	0.198
$\Delta = D/320$	1.359	0.686	0.198
Rosenfeld [10]	1.329	0.674	0.197

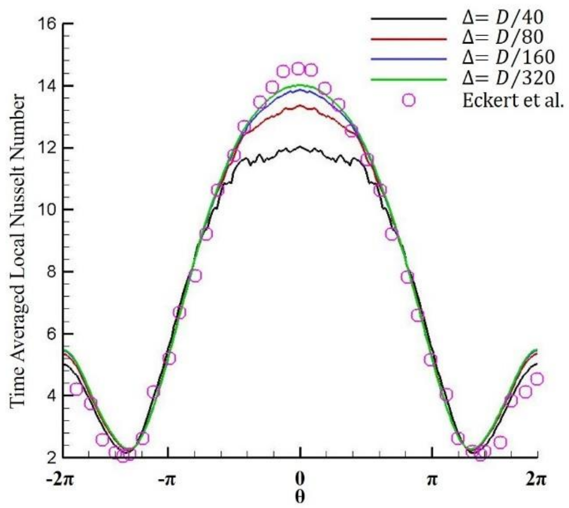

Figure 4. Comparison of time averaged local Nusselt number on single-resolution grid ($Re = 218$)

The present method is verified by comparing the local Nusselt number on the circular cylinder surface with the result on the single-resolution grid. In this study, in order to arrange the computational grid effectively, the dual-

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resolution grid is combined with the hierarchical Cartesian grid (shown Fig. 5). Figure 6 shows the time averaged local Nusselt number on the circular cylinder surface in $Re = 218$ on the single and dual-resolution grids. The local Nusselt number on the dual-resolution grid ($\Delta = D/80$-$D/160$) is closer to the single-resolution grid ($D/160$) than the single-resolution grid ($D/80$). The difference between it on dual-resolution grid and the single-resolution grid ($D/160$) appears because the velocity components for the advective term in the energy equation on the dual-resolution grid is interpolated.

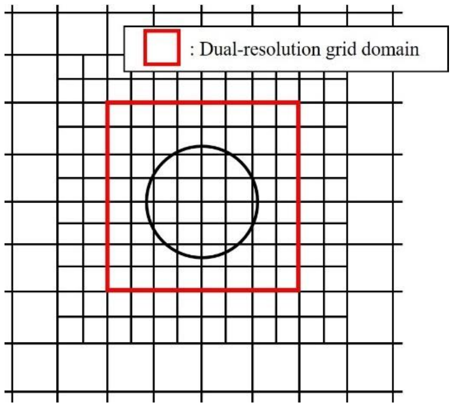

Figure 5. Computational grid combining dual-resolution grid and hierarchical grid

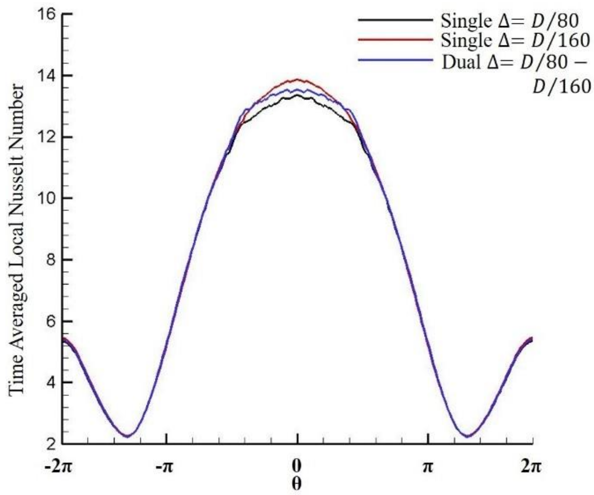

Figure 6. Comparison of time averaged local Nusselt number on single and dual-resolution grids ($Re = 218$)

Finally, the computational time in each grid is compared. In Table 2, the computational time ratio based on the single grid ($D/160$) to non-dimensional time $t = 200$ in is $Re = 200$ shown. The computational time is decreased by 60% in the dual-resolution grid compared with it in the single-resolution grid ($D/160$). And, the computational time in the dual-resolution grid is almost as it in the single-resolution grid ($D/80$). Therefore, it is concluded that the computational time for the flow simulation with heat transfer by using the SIBM is greatly reduced by introducing the dual-resolution grid method.

Table 2. Comparison of computational time ($Re = 200$)

	Computational time ratio
Single $D/80$	0.39

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Single $D/160$	1.00
Dual $D/80$-$D/160$	0.40

CONCLUSIONS

In this study, we proposed a simulation method combining the dual-resolution grid method and SIBM for the flow with heat transfer. First, we investigated the influence by the grid resolution on the results on the single-resolution grid for the flow around the 2-dimensional circular cylinder. As a result, it was confirmed that the required grid resolution was different between the result obtained by the Navier-Stokes equations and the result obtained by the energy equation. Therefore, the Navier-Stokes equations and the energy equation were solved at different grid resolutions by applying the present method combining the dual-resolution grid method. As a result, the Nusselt number obtained by the present method was close to the Nusselt number by the single-resolution grid with the same grid resolution for the energy equation. Furthermore, in the present simulations, the computational time in the present method was reduced by 60% compared to the single-resolution grid method with the same resolution for the energy equation because the Navier-Stokes equations are solved on a coarser grid than the energy equation. Therefore, it can be said that introducing the dual-resolution grid can greatly reduce the computation time for the flow simulation with heat transfer by using the SIBM regardless of the shape of the object. Finally, it is concluded that the present method combining the dual-resolution grid method and SIBM is very promising for the flow simulation with heat transfer involving objects with complicated shapes.

REFERENCES

[1] C.S. Peskin, D.M. McQueen. “A three-dimensional computational method for blood flow in the heart I. Immersed elastic fibers in a viscous incompressible fluid”. *Journal of Computational Physics*, Vol. 81, no.2, pp. 372-405, 1989.

[2] H. Nishida, K. Sasao. “Incompressible Flow Simulations Using Virtual Boundary Method with NewDirect Forcing Terms Estimation”. *Proceedings of International Conference on Computational Fluid Dynamics 2006*, pp. 185–186, 2006.

[3] M. Ishfaq, A.M. Khattak, G.A. Khan, Z. Anjum, Z. Ullah, R. Ullah, F. Jamal. “Soft B W-Hausdorff Space in Soft Bi Topological Spaces”. *Matrix Science Mathematic*, vol. 2, no. 2, pp. 28-31, 2018.

[4] L. Natrayan, E. Aravindaraj, M.S. Santhosh, M. Senthil Kumar. “Analysis and Optimization of Connecting Tie Rod Assembly in Agriculture Application”. *Acta Mechanica Malaysia*, vol. 3, no. 1, pp. 06-10, 2019.

[5] A. Abugalia. “Effect of Corona on The Wave Propagation Along Overhead Transmission Lines”. *Acta Electronica Malaysia*, vol. 3, no. 1, pp. 06-09, 2019.

[6] K. Rajput, A. Gupta, Arushi. “Re-Cycle of E-Waste in Concrete by Partial Replacement of Coarse Aggregate”. *Engineering Heritage Journal*, vol. 1, no. 1, pp. 05-08, 2019.

[7] K. Tajiri, H. Nishida, M. Tanaka. “Numerical Simulation of Incompressible Flows with Heat Transfer using Seamless Immersed Boundary Method”. *Journal of Computational Science and Technology*, Vol. 7, no. 2, pp. 286-296, 2013.

[8] V.H. Gada, A. Sharma. “On a Novel Dual-Grid Level-Set Method for Two-Phase Flow Simulation”. *Numerical Heat Transfer, Part B Fundamentals*, Vol. 59, pp. 26-57, 2011.

[9] H. Ding, C. Yuan. “On the Diffuse Interface Method Using a Dual-Resolution Cartesian Grid”. *Journal of Computational Physics*, Vol. 273, pp. 243-254, 2014.

[10] C.M. Rhie, W.L. Chow. “Numerical Study of the Turbulent Flow Past an Airfoil with Trailing Edge Separation”. *AIAA journal*, Vol. 21, no. 11, pp. 1525-1532, 1983.

[11] E.A. Fadlun, R. Verzicco, P. Orlandi, J. Mohd-Yosof. “Combined Immersed-Boundary Finite-Difference Methods for

International Journal of Mechanical Engineering Research and Technology

ISSN 2454 – 535X
www.ijmert.com
Vol. 1 Issue. 4, Nov 2013

Three-Dimensional Complex Flow Simulations". *Journal of Computational Physics*, Vol. 161, no. 1, pp. 35–60, 2000.

[12] K. Takeuchi, M. Tanaka, K. Sakuratani, K. Tajiri, H. Nishida. "A Conservative Level Set Method Using Dual-Resolution Grid". *Proceedings of 12th Asian Computational Fluid Dynamics Conference*, pp. 1–11, 2018.

[13] K. Kawakami, H. Nishida, N. Satofuka. "An open boundary condition for the numerical analysis of unsteady incompressible flow using the vorticity-streamfunction formulation (in Japanese)". *Transactions of the Japan Society of Mechanical Engineers Series B*, Vol. 60, no. 574, pp.1891–1896, 1944.

[14] M. Rosenfeld. "Grid refinement test of time-periodic flows over bluff bodies". *Computers & Fluids*, Vol. 23, no. 5, pp. 693– 709, 1994.

[15] E.R.G. Eckert, E. Soehngen. "Distribution of heat transfer coefficient around circular cylinder in cross flow at Reynolds numbers from 20 to 500". *Trans. ASME*, Vol. 74, pp. 343 –347, 1952.